The Upper Bounds of Graceful Rainbow Injection Arising from Combinatorial Nullstelensatz

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For a simple graph G, a graceful antimagic injection g in $[1, M]$ is a map $g : V(G) \rightarrow [1, M]$ such that the edge labels are different for all distinct edges and the vertex weights are different for all distinct vertices, where the edge label of uv is $f(uv) = |g(u) - g(v)|$ and the vertex weight of v is $\omega(v) = \sum_{uv \in E(G)} f(uv)$. If Y is a class of graphs of order n, not containing $\bigcup K_2$, $GA(Y, n)$ denotes the smallest integer M such that G admits a graceful antimagic injection on $[1, M]$. If $M = |V(G)|$, then G is called a graceful antimagic graph.

For $G_\Delta \subset Y$ a class of graphs with maximum degree Δ, we utilize Alon's Combinatorial Nullstelensatz to find an upper bound for $GA(G_\Delta, n)$, that is $GA(G_\Delta, n) \leq n+p(m-p)+\Delta(2n-5)$, where $p = \min\{\Delta, \lfloor \frac{n}{2} \rfloor\}$. We also prove that if $T_k \subset Y$ is a class of trees with k leaves, then $GA(T_k, n) \leq 3n - 3k$. In particular, a tree in T_k with $k = \frac{2n}{3}$ is graceful antimagic.

In this talk, we also introduce a variation of the previous notion of injection, that is the graceful $\mathbb{D}$-antimagic injection. Let $\mathbb{D}$ be a nonempty subset of the set $\{0, 1, \ldots, diam(G)\}$ and the $\mathbb{D}$-neighbourhood of vertex v, $N_{\mathbb{D}}(v)$, be the set of all vertices u that $d(u, v) \in \mathbb{D}$. A graceful $\mathbb{D}$-antimagic injection g in $[1, N]$ is a map $g : V(G) \rightarrow [1, N]$ such that all the edge labels are distinct and all the $\mathbb{D}$-vertex weights are also distinct, where the $\mathbb{D}$-vertex weight of v

is a sum of the labels of vertices in $N_{\mathbb{D}}(v)$. If X is a class of graph of order n with no two vertices with the same $\mathbb{D}$-neighbourhood, the smallest possible N such that every graph in X has a graceful $\mathbb{D}$-antimagic injection is denoted by $G\mathbb{D}A(X,n)$. If $N = |V(G)|$ then G is called a graceful $\mathbb{D}$-antimagic graph.

Let $\mathbb{D}$-degree of a vertex v be the cardinality of $N_{\mathbb{D}}(v)$. For $G_{\Delta_{\mathbb{D}}} \subset X$ a class of graphs with maximum D-degree $\Delta_{\mathbb{D}}$, we prove that $G\mathbb{D}A(X,n) \leq n + t(n-t) + p(m-p) - 1$, where $t = \min\{\lfloor \frac{n}{2} \rfloor, \Delta_{\mathbb{D}}\}$ and $p = \min\{\lfloor \frac{m}{2} \rfloor, \Delta\}$. If $T_k \subset X$ is a class of trees with k leaves, then $G\mathbb{D}A(T_k, n) \leq 4n - 4k$. In particular, a tree with $k = \frac{3n}{4}$ is graceful $\{1\}$-antimagic.

Keywords: graceful $\mathbb{D}$-antimagic, graceful antimagic, injection, combinatorial nullstelensatz

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