Counterexamples to the Total Vertex Irregularity Strengths Conjectures

Rinovia Simanjuntak

Combinatorial Mathematics Research Group
Institut Teknologi Bandung, Indonesia

For a positive integer k, the total vertex irregularity strength $\mathrm{tvs}(G)$ of a simple graph $G(V, E)$ is the smallest k so that there exists a function $\varphi : V \cup E \to [1, k]$ so that the vertex-weights (i.e., the sum of labels of a vertex and all of its incident edges) are all distinct. In 2010, Nurdin, Baskoro, Salman and Gaos posed two conjectures on the total vertex irregularity strength of trees and general graphs as follows: (i) for every tree T, $\mathrm{tvs}(T) = \max\{\lceil (n_1 + 1)/2 \rceil, \lceil (n_1 + n_2 + 1)/3 \rceil, \lceil (n_1 + n_2 + n_3 + 1)/4 \rceil\}$, and (ii) for every graph G with minimum degree δ and maximum degree Δ, $\mathrm{tvs}(G) = \max\{\lceil (\delta + \sum_{j=1}^{i} n_j)/(i + 1) \rceil : i \in [\delta, \Delta]\}$, where n_j denotes the number of vertices of degree j.

This talk provides infinite families of counterexamples to the afore-mentioned conjectures and proposes a novel conjecture on the total vertex irregularity strength of trees.

rino@math.itb.ac.id