Probabilistic aproach to conflict-free colorings

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A proper vertex colouring of a graph G is conflict-free if in the neighbourhood of every vertex some colour appears exactly once, while it is called h-conflict-free if there are at least h such colours for each vertex of G. The least numbers of colours in such colourings of G are denoted by $\chi_{\mathrm{pcf}}(G)$ and $\chi^h_{\mathrm{pcf}}(G)$, respectively. Using probabilistic methods, we show that $\chi^h_{\mathrm{pcf}}(G) \leq (1+o(1))\Delta$ if $\delta \gg \ln \Delta$ and $h \ll \delta$, and that $\chi_{\mathrm{pcf}}(G) \leq \Delta + O(\ln \Delta)$ for regular graphs. These are related with the conjecture of Caro, Petruševski and Škrekovski [1] that $\chi_{\mathrm{pcf}}(G) \leq \Delta + 1$ for every connected graph G of maximum degree $\Delta \geq 3$, towards which they proved that $\chi_{\mathrm{pcf}}(G) \leq \left\lfloor \frac{5\Delta}{2} \right\rfloor$ if $\Delta \geq 1$.

References

[1] Y. Caro, M. Petruševski, R. Škrekovski, Remarks on proper conflict-free colorings of graphs. arXiv preprint arXiv:2203.01088, 2022.

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