On uniqueness of packing of three copies of 2-factors

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Let G_1, G_2 and G_3 be three copies of a graph G of order n. We say that there exists a packing of three copies of G into a complete graph K_n if there exist injections $\alpha_i : V(G_i) \to V(K_n)$, $i \in \{1, 2, 3\}$, such that, for $i \neq j$, $\alpha_i^*(E(G_i)) \cap \alpha_j^*(E(G_j)) = \emptyset$, where the mapping $\alpha_i^* : E(G_i) \to E(K_n)$ is induced by α_i. Two packings of three copies of G are distinct if the graphs $\alpha_1(G) \oplus \alpha_2(G) \oplus \alpha_3(G)$ and $\alpha_1'(G) \oplus \alpha_2'(G) \oplus \alpha_3'(G)$ are not isomorphic. We say that a packing of three copies of G is unique if all packings of three copies of G are isomorphic.

Let $C_{n_1} \cup C_{n_2} \cup \ldots \cup C_{n_k}$ be a 2-factor *i.e.* a vertex-disjoint union of cycles. We completely characterize 2-factors *i.e.* we present which 2-factors do not have packing of three copies, which have unique packing of three copies and which have at least two distinct of three copies. This problem is a generalization of the problem of uniqueness of packing of two copies of a 2-factor which was proposed and solved by Grzelec, Pilśniak and Woźniak in 2023.

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